

**INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE**  
**B.MATH - Second Year, 2005-06, Statistics - I**  
**Midterm Examination, 15 September 2005**  
**Calculator and Statistical Tables may be used**  
**Marks are shown to the left of each question**

(10) 1. The number of eggs ( $X$ ) laid by an insect can be assumed to have the Poisson distribution with unknown mean  $\lambda$ . Once laid, each egg has an unknown chance  $p$  of hatching, and the hatching of one egg is independent of the hatching of the others.

- (a) What is the probability distribution of the number ( $Y$ ) of hatched eggs of this insect?
- (b) What is the probability distribution of the number ( $Z$ ) of eggs of this insect which are not hatched?
- (c) What is the conditional probability distribution of  $X|Y = y$ ?

(10) 2. Let  $X \sim \text{Poisson}(\lambda)$ . Define  $X_T$  by  
 $P(X_T = x) = P(X = x)/P(X > 0), \quad x = 1, 2, \dots$   
Find the mass function, mean and variance of  $X_T$ .

(10) 3. Forty nine measurements are recorded to several decimal places. Each of these measurements is rounded off to the nearest integer. The sum of the original 49 numbers is approximated by the sum of these integers. If we assume that the round-off errors are independent  $U(-\frac{1}{2}, \frac{1}{2})$ , compute approximately the probability that the sum of the integers is within 2 units of the true sum.

(10) 4. Let  $Y \sim \text{Exponential}(\lambda)$  with p.d.f.  $f(y) = \lambda \exp(-\lambda y), y > 0$ .  
Define  $X$  by  $X = k$  if  $k - 1 \leq Y < k, k = 1, 2, \dots$ . Consider a random sample  $X_1, X_2, \dots, X_n$  from  $X$ . What is the sampling distribution of  $\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$ ? Is  $\bar{X}$  a consistent estimator of  $\frac{1}{\lambda}$ ?

(10) 5. Let  $X_1, X_2, \dots, X_n$  be a random sample from  $U(0, \theta)$ , where  $\theta > 0$  is unknown.

- (a) What is the probability distribution of  $X_{(n)}$ , the largest order statistic?
- (b) Show that  $X_{(n)}$  is a consistent estimator of  $\theta$ .